To: J. Onken

From: F. VanLandingham

Date: May 14, 1996

Subject: Determination of the AXAF IRU Scale Factor / Alignment

TRW performed a study (Reynolds, R. G., "Evaluation of OFLS Scale Factor / Misalignment Calibration", August 8, 1995) of the OFLS specified algorithm (AXAF OFLS Software Design Specification, AMO-2310, September, 1995) for the operational determination of the AXAF IRU scale factor / alignment. The results of this study produced an examination of the method for acquiring the required observational data and the validity of the specified algorithm. This examination was performed by the Mission Planning Working Group (MPSWG). The OFLS accepted an action from the MPSWG to summarize the examination and to describe a method for providing an acceptable algorithm. The enclosed memorandum is the response to this action.

cc:

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OFLS Computation of the IRU Scale-Factor / Alignment

Problem: TRW proposed that during the mission, the AXAF-I IRU scale-factor / alignment will need to be updated at least once every two weeks, preferably once every week.

Mission Impact: This would impose an additional constraint on the planning of the science schedule.

OFLS Impact: This constraint could be accommodated in the OFLS Mission Planning and Scheduling (MPS) subsystem. In addition, TRW proposed that the OFLS Attitude Determination and Sensor Calibration (AD&SC) subsystem should determine the attitude and IRU calibration parameters based on a Kalman filter and not the currently baselined deterministic method.

OFLS Assessment: Considering information gathered to date, OFLS plans to make no modifications to the MPS. Similarly, OFLS has no plans to modify the IRU calibration algorithm. OFLS will study the applicability of the implemented algorithm and, if necessary, determine any needed modifications.

This paper provides a summary of the events and reasons that lead to this assessment.

INTRODUCTION

The accuracy requirement for an AXAF-I maneuver is such that the worst case error after an 180-degree maneuver is 100 arcsecond (3σ) (Reference 1). (Note: Section 3.7.5.2.1.7 of reference 2 states the requirement as 120 arcsecond (3σ) after a 90 degree maneuver). The ability to achieve this requirement is dependent on the accuracy of the scale-factor / alignment used by the onboard pointing control system in performing the maneuver.

For AXAF, the IRU scale-factor / misalignment is to be updated based on a redetermination using algorithms installed in the OCC's OFLS. TRW undertook an analysis (Reference 1) of the algorithm specified and baselined for this computation (Reference 3). Based on this analysis, TRW concluded that the OFLS "algorithm can produce scale-factor calibration errors as large as 800 ppm and misalignment errors as large as 250 arcsecs (3σ)". To achieve the maneuver accuracy requirement, TRW made two recommendations:

1. for monitoring and determining the scale-factor / misalignment, modify the OFLS algorithm to use a Kalman filter or, at a minimum, change the weighting scheme in the current algorithm
2. to maintain an accurate scale-factor / misalignment, update the onboard value at least once every two weeks. Subsequently, TRW indicated that once every two months is sufficient (Reference 4)

Implementation of these recommendations in the OFLS has two major impacts - scheduling of science observations as well as the modification to the baselined IRU calibration algorithm.

OBSERVATIONAL DATA NEEDS

The determination of the IRU scale-factor / alignment requires observations that compare the maneuver size and direction as measured by the IRUs with the size and direction as measured by a reference, which for AXAF are ACA star observations. To maneuver in three dimensions requires observational maneuvers about the three axes.

The number of maneuvers required to achieve the accuracy required for AXAF can be estimated as a function of the maneuver size. Reference 1 states that the driving error uncertainty is the pre- and post-maneuver attitude roll error. This is caused by the limited field-of-view of the aspect camera. Reference 1 estimates this uncertainty to be 16 arcseconds (1σ). This corresponds to a maneuver uncertainty of $3*\sqrt{2}*16 = 70$ arcseconds (3σ). For a set of N statistically independent maneuvers of size θ degrees, the maneuver calibration parameters (i.e., scale-factor / alignment) may be determined to an accuracy of approximately $70/(\theta*\sqrt{N})$ arcseconds. The AXAF performance requirement is 100 arcseconds per 180 degrees (3σ). Thus $N = [\ (70 * 180) / (\theta * 100)\ ]^2 = (126 / \theta)^2$. For example, for statistical accuracy, a minimum number of two 90 degree maneuvers is required. For observability, one maneuver about each spatial axis is required.

PLANNING and SCHEDULING

The observational maneuver data can be obtained either as a by-product of maneuvers scheduled to satisfy the science program or by interrupting the science program and performing the requisite maneuvers.

Incorporate into science planning - The former can be implemented by imposing a requirement on the OFLS MPS subsystem to plan maneuvers of sufficient size, direction, and frequency. This would necessitate the specification and implementation of an additional condition in the planning of the science schedule. This condition would need to consider:

1. stability of the scale-factor / alignment
2. characteristics of the planned maneuvers.

Stability of the scale-factor / alignment - The need to determine a new value of the scale-factor / alignment is determined by comparing the lengths of two time intervals:

1. time interval since the last determination
2. estimated time interval in which the scale-factor / alignment would change sufficiently to produce errors exceeding the requirement.

The expected time interval, T, between calibrations can be estimated as a function of maneuver size. Reference 4 states that the IRU scale-factor end-of-life stability is 425 ppm. The expected life of the IRU is a total of 9.6 years (5 years for on-orbit operation, 4 years for storage, 5000 hours for ground operation). If the IRU conforms to specification, then the maximum error due to change in the scale-factor is 425 ppm *

$\sqrt{(T/9.6)}$ To successfully find acquisition and guide stars, the error after a maneuver of θ degrees is 100 arcseconds or 100 arcseconds / θ degrees. Equating the two expressions and solving for T, we find that $T = 41010/\theta^2$ years. For a maneuver of 30 degrees, then T = 46.6 years; for 60 degrees, T = 11.4 years; for 90 degrees, T = 5.1 years; and for T = 180 degrees, T = 1.3 years.

Characteristics of the planned maneuvers - Suppose the scale-factor varies on a time scale that permits acquisition of the required observational data, computation and operational utilization of the observed value, then the number of required maneuvers can be estimated. If the size of the maneuvers being planned is larger than the maximum size of the maneuvers used for the latest determination, then the current value may not be adequate. As shown above, the number of maneuvers required for determining the scale-factor to a required accuracy is a function of the size of the maneuvers used for the observation. For example, suppose the value of the scale-factor is based on a series of 30 degree maneuvers. To achieve the same accuracy, nine times as many 30 degree maneuvers are needed as when using 90 degree maneuvers. Thus, the number of available 30 degree maneuvers may be insufficient for determining a scale-factor value that is accurate enough to satisfy the desired error for a planned 90 degree maneuver.

The OFLS presented to the Mission Planning and Scheduling Working Group (MPSWG) its concerns about the impact of imposing a scale-factor / alignment condition on the MPS. The MPSWG decided that no such imposition is required. Instead, the MPSWG decided that the following operational approach would be used. The Flight Operations Team (FOT) would review the mission schedule to determine if the appropriate maneuvers are included. If so, no additional revisions to the schedule would be required. If not, the FOT in consultation with the Science Operations Team (SOT) would revise the science schedule in a manner that would produce the appropriate maneuvers. Thus, the FOT in cooperation with the SOT would assume the responsibility for scheduling the observational maneuvers necessary for the determination of the IRU scale-factor / alignment.

Use a specific operational sequence - The science schedule is interrupted and the spacecraft is commanded to perform a defined set of maneuvers that provide the required observational data. To maintain a high efficiency for science data acquisition, this method can be used only if the scale-factor / alignment value changes slowly. According to the above estimate, it is anticipated that this would be performed approximately once a year, provided the IRU performs according to specification.

SCALE-FACTOR / ALIGNMENT DETERMINATION

The algorithm specified by the OFLS conforms to the approach taken on previous missions. The scale-factor / alignment is measured during orbital verification (OV) and monitored throughout the mission. The need for a redetermination is evident from a systematic increase in the size of the offset of the target from the expected location in the FOV, especially after large maneuvers. On previous missions, the rate of increase in the offset is gradual and does not prohibit the acquisition of the target. However, it may increase the amount of time required to place the target at the desired location in the FOV.

Performance of the OFLS Algorithm - The graphs on pages 22 - 34 of reference 1 provide a review of the accuracy of the OFLS algorithm. The graphs, pertaining to OFLS performance under various assumed conditions, represent recalibrations using one week's worth of operationally available maneuvers. The available maneuvers are based on the selected "typical" maneuver distribution. The noisy quality of the graphs indicates that for some weeks there is a good distribution of maneuver types for calibration purposes (i.e., good visibility in pitch, roll, and yaw), whereas for other weeks the distribution is poor and the calibration results are correspondingly poor. (Unfortunately, the paper presents histograms characterizing the maneuver distribution but does not provide details for the maneuvers for each week.) For an optimally selected set of calibration maneuvers, the OFLS algorithm will produce results close to -- maybe even below -- the bottom of these curves for each set of conditions.

Under "ideal conditions" - i.e., no gyro noise or instability - (Figure 5.3 page 22), the expected error after a 180 degree maneuver would be about 60 arcseconds. Based on the error budget used in reference 1, Figure 5.9 on page 28 is the most relevant for the current OFLS implementation. In this case a calibration based on an optimally selected set of maneuvers, errors after a 180 degree maneuver of order 200 arcseconds is predicted. This raises a valid concern by indicating that the current OFLS algorithm would produce results incompatible with the maneuver requirements. Consequently TRW suggests the implementation of one of two modifications

1. implement a Kalman filter
2. modify the current OFLS weighting scheme

The study results indicate that a Kalman filter approach is not necessary. Figure 5.12 on page 31 shows a comparison of the modified OFLS performance vs. Kalman filter performance. When the maneuver distribution is good, e.g., around day 40, the two methods have comparable accuracy -- around 35arcseconds after a 180 degree maneuver.

The need to modify the OFLS weighting scheme is inconclusive. Based on our review of the study, we have several concerns about the assumptions and the study results. These are summarized in the first attachment. We have reviewed TRW's suggested weighting scheme and, based on our concerns, have proposed alternatives (described in the attachment). These alternatives are the result of preliminary analysis. We have no conclusive evidence that *any* of these schemes describe the required modification. Consequently, the OFLS will proceed as follows. We will implement the IRU calibration algorithm as it is currently specified (Reference 2). Using this implementation as well as the OFLS attitude simulator and attitude determination functions, we will perform a study to verify its applicability. The results will be analyzed to determine if a modification is required. If a modification is necessary, the OFLS will use the study results to determine the most appropriate one.

SUMMARY

The FOT will be responsible for monitoring the accuracy of the IRU scale-factor / alignment. The FOT will determine when a new value must be determined and will schedule as necessary maneuvers to acquire the observational data.

No modifications to the OFLS MPS to include a scheduling criterion for gathering observational data for the determination of the IRU scale-factor / alignment will be made.

IRU specifications indicate that an operationally-determined value for the scale-factor / alignment should not produce a violation of the maneuver requirement over a timescale of approximately one year. The OFLS algorithm is based on the acquisition of data from appropriate spacecraft maneuvers. The specifications for the AXAF IRUs and maneuver accuracy are comparable to those used on other spacecraft in which this technique has provided adequate support. However, OFLS will study the implemented algorithm to determine its suitability and, if necessary, any modifications that are necessary.

The determination of the scale-factor / alignment should be a primary objective of the orbital verification (OV) phase. While it will be monitored throughout the mission, a study of the stability of the scale-factor / alignment should be a part of OV and the early mission.

REFERENCES

1. Reynolds, R. G., “Evaluation of OFLS Scale-factor / Misalignment Calibration”, August 8, 1995
2. AXAF-I Spacecraft Subsystems Specifications, Baseline Review, 18 July 1995
3. AXAF OFLS Software Design Specification, AMO-2310, September, 1995
4. “Error Budget for 180 degree Maneuver Error”, Fax from S. Shawger to Mission Planning and Scheduling Working Group meeting, 3/27/96
5. AXAF-I Pointing Control Analyses, SE 11k Volume 2, Section 6.0

To: F. VanLandingham

From: G. Welter

Date: 29-April-96

Subject: Comments on Gyro Errors and Algorithms

This memo provides some comments based on your request to assess the implications for Reid Reynolds's August 8, 1995 evaluation of the OFLS gyro calibration algorithm resulting from Steve Shawger's list of basic input errors. Upon reviewing Reid's memo, I find I have two areas of concern about his conclusions: Reid's understanding of the attitude solutions produced by QUEST and the attitude accuracies he assumed.

QUEST Attitude Solutions: Although we've been concentrating on the issue of adjusting the weighting scheme for the input data, Reid's claim that the baseline OFLS algorithm is inadequate is actually as much dependent on his understanding that the OFLS algorithm will be using "one shot QUEST update" with a 1-sigma roll uncertainty for the attitude solutions of 0.16 arc-second in pitch & yaw and 16 arcsecond in roll. (See page 11 of his memo.) Reid claims that this is an unnecessarily large attitude error, and that using the OBC Kalman filter for attitude data smoothing, or some equivalent ground processing of data, will result in attitude solutions that scale like $(\sigma_m \sigma_v T^{1/2}/N)^{1/2}$, where σ_m is the aspect camera measurement error, σ_v is the gyro angle random walk, T is the OBC Kalman filter update, and N is the number of stars. This scales relative to his estimate for the "one shot QUEST" attitude accuracy by $(\sigma_v T^{1/2}/\sigma_m)^{1/2} \sim 0.34$. Ignoring for the moment that I don't agree with his estimates for the attitude accuracies, but assuming that he did his other calculations correctly, my understanding of his paper is that the difference in pre- and post-maneuver attitude accuracy accounts for the differences between his Figures 5.9 (Current OFLS Performance with All Errors) and Figure 5.11 (Without Modified Weighting, Using On-Board Attitude).

Looking at the lower curve envelopes for the two figures, the rescaling by 0.34 estimated in the previous paragraph actually works pretty well; there is about a factor of three improvement in calibration accuracy when using attitudes with accuracies that are better by a factor of three. Let's assume for the moment that Figure 5.11 is correct for solutions using optimally computed attitudes. Given that most of the time, the Figure 5.11 solutions meet the 100 arcsecond per 180 degree requirement, and further that the difference between the solutions is driven by differences in available maneuvers for use in the calibration, one is tempted to conclude that the OFLS algorithm would be acceptable in conjunction with specially selected maneuvers if optimally computed attitudes were being used. This leads to a question: what makes Reid conclude that the OFLS algorithm is using non-optimal attitude solutions? Phrased another way, what does Reid mean by a "one shot QUEST" solution, and how does it deviate from optimal? After reading some other AXAF documents, I am left wondering whether Reid thinks the one-shot solution pertains to that derivable with acquisition stars, as opposed to guide/aspect stars. Operationally, the analyst should be careful to make sure the guide stars are being used for the post-maneuver attitude solution.

Attitude accuracy estimates: I have two concerns pertaining to Reid's attitude accuracy estimates. First, I disagree with his estimate for the pitch & yaw attitude accuracy from a Kalman filter (or any other filter) implied by his equation 3.10. Second, I disagree with his estimate for γ, although this is of lesser concern. With respect to the first concern, note that using $(\sigma_m \sigma_v T^{1/2}/N)^{1/2}$ as the pitch & yaw accuracy implies that either perfect gyros (σ_v=0) or an infinitely fast Kalman filter (T=0) would result in an error free attitude estimate, despite nonzero star measurement errors. I suggest instead something like the following equation as appropriate for pitch, yaw, and roll uncertainty covariances (I have ignored the leading factor of 1/4 in equation 3.10; that is associated with the use of half angles in quaternions):

$$U_{P,Y} = \sigma_C^2 + (\sigma_c^2 + \sigma_m^2 + \sigma_v^2 T_W) / N$$

$$U_R = 2 (\sigma_c^2 + \sigma_m^2 + \sigma_v^2 T_W) / N \sin^2(\gamma)$$

where σ_C is the low-frequency catalog uncertainty, σ_c is the high-frequency catalog uncertainty, σ_m is the intrinsic measurement error associated with Aspect Camera observations, σ_v is the gyro angle noise, T_W is the half width of the attitude window, and γ is the scaling angle for roll uncertainty. Reid ignored the catalog errors. The low frequency catalog error systematically cancels out of roll estimates, which is why I left it out of the expression for U_R. As noted earlier, I happen to disagree with Reid's estimate for γ. He has γ=1.4 deg / $6^{1/2}$ = 0.57 deg. I estimate something close to 1.0 degree. My back of the envelope estimate for γ takes into account the fact that roll estimates should give higher weight to star pairs with larger separations. I'll use my estimate below.

Steve Shawger's estimates for three of the input error values are σ_c ~ .2", σ_m ~ .2", and σ_v~ .02"/sec$^{1/2}$. The latter implies that the window would have to be about 200 seconds long in order for the gyro error to become the largest of the three. In a conversation that I had with Conrad Sturch (head of the Guide Star Selection Section at the Hubble Space Telescope Science Institute.) yesterday, Conrad suggests $3\sigma_C$ ~ 1" and $3\sigma_c$ ~ .5". (Actually, Conrad did not recall whether the values were for one or three sigma; I've assumed the latter since that makes Steve's estimate for σ_c agree reasonably with Conrad's.) For gyro calibration purposes, which involve maneuvers between uncorrelated parts of the sky, the pitch & yaw uncertainties are going to be dominated by the low-frequency catalog uncertainties; this is even more true if Conrad's numbers are 1-sigma. Using these input error estimates, and assuming 5 stars and T_W ~ 100 seconds, I find the error uncertainties to be

$U_{P,Y}^{1/2}$ ~ 0.4 arcseconds (Gary' estimate)

$U_R^{1/2}$ ~ 12 arcseconds (Gary' estimate)

which contrasts with Reid's two estimates:

$U_{P,Y}^{1/2}$ ~ 0.16 arcseconds (Reid's one-shot QUEST estimate)

$U_R^{1/2}$ ~ 16 arcseconds (Reid's one-shot QUEST estimate)

$U_{P,Y}^{1/2}$ ~ 0.054 arcseconds (Reid's "optimal" [Kalman] estimate)

$U_R^{1/2}$ ~ 5.4 arcseconds (Reid's "optimal" [Kalman] estimate)

My conclusion is that although Reid obtained good results for the lower envelope in Figure 5.11, you can't trust them to be representative of what the baseline OFLS software will yield when provided with true attitude errors, nor is Figure 5.10 a good estimate of how well the OFLS software would work with modifications as proposed by Reid. His estimate of the true attitude knowledge possible from the data is too good. (This also means that Reid's estimate for how well his Kalman filter approach would work [Figure 5.12] is too optimistic.)

The preceding conclusion is unfortunate. Given that my attitude error estimates are closer to Reid's "one-shot" estimates than his "optimal" estimates, I'd conclude that the closest of Reid's figures to what I'd predict for the baseline OFLS algorithm (assuming proper attitude errors) is Figure 5.9, which shows a lower-envelope performance a factor of two worse than the requirement. If we assume that we can scale the results based on the ratio of the roll uncertainty (a problematic assumption when improper weighting is being used), we might hope for something like a 25 percent improvement in performance relative to Figure 5.9 -- still well shy of the requirement.

Let's bite the bullet and allow for more appropriate weighting. I think you can reasonably argue that Reid's estimate for variability time scales based on his random walk model for scale factor and alignment changes are not likely to be realistic. Reid had claimed, based on the size of σ_ε in his Table 3.2, that alignment changes would dominate the problem. Steve Shawger's numbers put a 3-sigma uncertainty for the alignment error as ~ 18", not explicitly time dependent. (I understand from Fletcher Kurtz, the MSFC engineer responsible for the on-orbit calibration of the HEAO-II gyros, that the AXAF alignment variation is expected to be thermally driven, with an 18" limit.) From an algorithmic perspective, the elimination of the short time scales for variation means we really do not have to worry about the complications of Kalman filtering or modifications to the OFLS algorithm intended to solve the same problem. In particular, we can ignore both inclusion of an a-priori estimate with associated weight as well as terms designed to reduce the weight as a function of time between the maneuver and the calibration epoch (i.e., Reid's proposed last term in equation 3.9). I propose the following equation:

$$\boldsymbol{W}_n^{-1} = \boldsymbol{A}_n \boldsymbol{P}^i{}_n \boldsymbol{A}_n^T + \boldsymbol{P}^f{}_n + \sigma_{U1}^2 \boldsymbol{I} + \sigma_{U2}^2\, \Theta_n^2 \boldsymbol{I}$$

where n is the maneuver number, the $\boldsymbol{P}$ matrices are the spacecraft frame attitude covariance matrices, $\boldsymbol{A}$ is the pre- to post- maneuver transformation matrix, $\boldsymbol{I}$ is the identity matrix, Θ is the maneuver angle, and σ_{U1} and σ_{U2} are user provided parameters. The first two terms, $\boldsymbol{A}_n \boldsymbol{P}^i{}_n \boldsymbol{A}_n^T$ and $\boldsymbol{P}^f{}_n$, are simply the attitude covariances that the OFLS software already includes. Parameter σ_{U1} is used to absorb error terms that are essentially constant and large relative to the pitch and yaw attitude uncertainties. Based on Steve's error table, σ_{U1} will be dominated by contributions from the IRU to AC temperature-dependent alignment variation; setting it to about 6" would be reasonable. Parameter σ_{U2} is used to absorb errors that depend on the size of the maneuver. Based on Steve's error table, σ_{U2} will be dominated by scale factor asymmetry and uncompensated nonlinearity; setting it to about 17 ppm (i.e., ~ 11" for a 90 degree maneuver) would be reasonable. Note that, for maneuvers near 90 degrees, the latter error is about the same size as my estimate for the attitude roll uncertainty. Operationally, the user would adjust the values of σ_{U1} and σ_{U2} to prevent any residuals from being excessive.

In my opinion, the weighting scheme provided above captures the dominant aspects of the problem. With good values for the adjustable parameters, it should transform calibration performance from that associated with the bottom envelope of Reid's Figure 5.9 to his Figure 5.7. Actually, with reasonable weighting taken into account, I'd even be willing to predict some degree of improvement based on the reduction of roll uncertainty from ~ 16" to ~ 12". In a back of the envelope sense, I estimate performance based on worst case values for each of the contributing terms to the maneuver covariance (i.e., inverse weight) matrix, leading to a 180 degree maneuver error of ~ 90 arcseconds. Using 16" roll errors instead of 12", I find ~ 96" from the same calculation. The change between the two is not very much; within the spirit of the calculation, both agree pretty well with the lower envelope of Reid's Figure 5.7.

I find that the algorithm, even with the adjusted weighting, will produce results close to the requirement limit. The best (and perhaps only) way to drive it down further (if there is a perceived need to do so) would be to attack the dominant error source, which is that associated with scale factor asymmetry and nonlinearity. If the errors are systematic, it should be possible to reduce them with a calibration scheme like that in the paper I sent you. Personally, I would not recommend developing software for that purpose at this time. It may be worth revisiting that issue after launch if at that time you find that the gyros are performing inadequately.